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# Incentive mechanism for selfish nodes in wireless sensor networks based on evolutionary game

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ABSTRACT

A Wireless Sensor Network (WSN) is made up of a mass of nodes with the character of self-organizing, multi-hop and limited resources. The normal operation of the network calls for cooperation among the nodes. However, there are some nodes that may choose selfish behavior when considering their limited resources such as energy, storage space and so on. The whole network will be paralyzed and unable to provide the normal service if most of the nodes do not forward data packages and take selfish actions in the network. In this paper, we adopt a dynamic incentive mechanism which suits wireless sensor networks based on the evolutionary game. The mechanism emphasizes the nodes adjust strategies forwardly and passively to maximize the fitness, making the population in the wireless sensor network converge to a cooperative state ultimately and promoting the selfish nodes cooperating with each other such that the network could offer normal service. The theoretical analysis and simulation results show that the proposed model has better feasibility and effectiveness.



## 1. Introduction

Sensor Networks consist of a large number of tiny sensor nodes with the advantage of low cost and power consumption which are deployed in a certain region. Wireless sensor networks have the characteristics of being self-organizing, multi-hop and decentralized. Therefore, they have been widely used in military, traffic, industry and agriculture and so on. Since there is no fixed infrastructure and unified control center in wireless sensor networks, the data transmission needs cooperation among the nodes. However, so far, the nodes in the network are always supposed to have a property of favorable cooperation and provide service to others for free. As the nodes are affected by the limitation of energy, storage space and computational capabilities, there may exist many selfish nodes to some extent. Therefore, this assumption does not always stand in a practical wireless sensor network environment, especially when the nodes belong to many different organizations. Obviously, the whole network will be paralyzed and unable to provide normal service if most of the nodes take selfish actions and forward data packages in the network. Therefore, one of the important challenges in the research on wireless sensor networks is how to motivate the selfish nodes to cooperate and ensure the normal operation in the network.

There are two approaches to solve the selfish problem in wireless sensor networks. One is based on the reputation mechanism, the other is on a payoff mechanism [1,2]. The main model predicts the behavior of the nodes based on conventional game theory [3]. However, the conventional game theory analyzes the problems based on the fully rational assumption which calls for all participants to have many perfect requirements such as rational consciousness, analysis reasoning ability, recognizing ability, memory, accurate behavior ability and so on. Conventional game theory emphasizes

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that the participants never make mistakes at any time. Meanwhile, it is necessary to trust others during the game and to pay more attention to the results of static equilibrium.

There are several disadvantages in the previous incentive mechanism model: firstly, it could not describe the strategies of the dynamic evolution process for the nodes accurately. Moreover, it is hard to ensure the robustness and stability of these mechanisms they lack strict mathematical theory analyses; secondly, they assume that each node has to own and keep the global information in most of the current mechanisms. Therefore, the nodes must have some resources like strong cognitive ability and storage space. It is obvious that this assumption is unrealistic under normal circumstances in wireless sensor networks. Thirdly, these mechanisms cannot optimize the fitness of each node even though they can ensure the optimal performance of the system. Hence, the nodes still have probability of selfish behavior. Qiu showed that the model can defend actively in the wireless sensor network based on the evolutionary game theory [4].

The traditional network security schemes have provided protection against malicious parties. However, the open and anonymous nature of the wireless sensor network makes them vulnerable to rational nodes. Therefore, we argue that the traditional mechanism such as authentication, access control and social control mechanisms [5,6] are typical examples of reputation system. Evolutionary game theory describes how successful strategies spread in a population. Specifically, such an approach generally includes three phases: interaction phase, mutation phase [7–9]. Interaction in the wireless sensor network means that the nodes model the games like Prisoners Dilemma or cooperation [10]. Mutation means that the nodes in the wireless sensor network change their strategies because of more optimal fitness or less resources will be used. At last, we consider the traditional evolutionary game dynamics of all these three strategies as given by the replicator equation [11] and absorb the results.

In this paper, we propose a dynamic incentive mechanism for wireless sensor networks based on the evolutionary game. The evolutionary game gives up the fully rational hypothesis on a conventional game, which emphasizes the limited rationality of the participants and the dynamic evolution during the game procedure. It means that the nodes only need a part of information about the game states of the whole network instead of the global states of the network. These states include the fitness of the strategy. The nodes keep on learning and imitating during the game. Also, through trial and error they can find out the best strategy which maximizes their own fitness. By doing these, the nodes which take selfish actions are rejected from the network and the whole network will reach a cooperative state at last. Thus, we apply the strict mathematical theory of knowledge to prove the stability of the system and the simulation tool to validate the correctness of the theoretical analysis.

## 2. Incentive mechanism organization

In general, an evolutionary game model should include the following components: a number of the population, a set of strategies, a stage game in normal form and a dynamic adjustment process. At every moment, the stage game specifies the expected fitness of each strategy, and the state specifies the current distribution of strategies employed in each population [12]. However, the fitness of different strategies also changes as the state changes over time. Thus, the time path could be rather complicated. Note that the analysis focuses mainly on steady states. In the following subsections, we briefly introduce the necessary components which are related to our model.

### 2.1. Description of selfish nodes

The wireless sensor network provides a normal network through the cooperation of the nodes. However, the nodes need some resources such as energy and storage during the cooperation. Therefore, the nodes may choose the selfish non-cooperative behavior to think of the limited resources for themselves. It is obvious that it will influence the performance of the whole network seriously. In order to analyze the selfish problem of the nodes effectively, we make some assumptions for wireless sensor networks as follows:

(1) The wireless sensor network consists of $N$ nodes, each node should have the routing forwarding function.
(2) Each node has two alternative available strategies. One is a cooperative strategy to forward data packages, the other is a non-cooperative strategy.
(3) If the nodes both choose a cooperative strategy, they will gain $R$ units of payoff and consume $C$ units of energy; if the nodes both choose a non-cooperative strategy, they will gain 0 units of payoff; if one choose a cooperative strategy and the other one chooses a non-cooperative strategy, then the cooperative one will gain 0 units of payoff and consume $C$ units of energy while the non-cooperative one will gain $R$ units of payoff.

Therefore, we can obtain the payoff matrix from the assumption which shown as follows:

$$\begin{array}{cc} & \begin{array}{cc} C & NC \end{array} \\ \begin{array}{c} C \\ NC \end{array} & \left[\begin{array}{cc} R-C & -C \\ R & 0 \end{array}\right] \end{array}. \tag{1}$$

From the payoff matrix we can see that the forwarding package process of the nodes is a typical prisoners' dilemma problem. If all the nodes choose non-cooperating at last, then it leads to the paralysis of the network.

### 2.2. Assumption

In this paper, we provide a new cooperative strategy based on the reciprocity condition to motivate the nodes to cooperate. Now we give some assumptions for the incentive game model:

(1) Players: $N$ nodes in the wireless sensor network form a population. The number of the population $N \geq 1$ represents the number of different roles in the model, where each population has two or more alternative strategies available to its members. In the evolutionary game, the number of populations can be reduced to $N = 1$ by symmetrizing the game. In a symmetric game, all nodes have the same strategy set and payoff matrix. The payoff of a strategy depends on the strategies adopted by the others, but not on who is playing it.

    Note that $N$ represents a large but finite population. Recently, there are many works on evolutionary game dynamics in a finite population size. A stochastic description is necessary in a finite population.

(2) Strategy space: each node has three strategies, ALLC, ALLD and R. The strategic behaviors of ALLC, ALLD and $R$ are described respectively as follows: ALLC users provide service all the time; ALLD users never provide service; and through distinguishing the reputation of each node, $R$ employers always provide service to other ALLC and $R$ users, and do not provide service to ALLD users. Naturally, $R$ users have to pay seeking cost $C_R$ for information seeking which we call the reciprocation cost.

There is a fixation probability of a strategy: the probability that a single mutant strategy overtakes a homogeneous population which uses another strategy. When only including two strategies, the strategy with higher fixation probability is considered to be more "prosperous" by selection. We consider frequency-dependent selection in a single and well-mixed population of $N$ individuals, in which there exist two general types of individuals, $i$ and $j$, and individuals interact with others in pairwise encounters.

### 2.3. Fitness payoff matrix

For simplicity, we only deal with the stage game in normal form. Without loss of generality, we consider a game with $n$ strategies. Thus the payoff values are given by an $n \times n$ payoff matrix $A = [a_{ij}]$. The entry $a_{ij}$ means that a node gains payoff $a_{ij}$ by using strategy $i$ while interacting with a node who uses strategy $j$.

According to the behaviors of different strategies, the payoff matrix $A$ could be given as follows, where $C$ is the cost, $C_R$ is the small cost for determination.

$$\begin{array}{c} \\ \scriptstyle ALLC \\ \scriptstyle ALLD \\ \scriptstyle R \end{array} \begin{array}{c} \begin{array}{ccc} \scriptstyle ALLC & \scriptstyle ALLD & \scriptstyle R \end{array} \\ \begin{bmatrix} R-C & -C & R-C \\ R & 0 & 0 \\ R-C-C_R & -C-C_R & R-C-C_R \end{bmatrix} \end{array}. \tag{2}$$

The pairwise comparison of the three strategies leads to the following conclusions:

(1) ALLC is dominated by ALLD, which means that it is best to play ALLD against both ALLC and ALLD.
(2) $R$ is dominated by ALLC. These two strategies cooperate with each other, but the complexity cost of $R$ implies that ALLC has a higher payoff.
(3) If the average number exceeds a minimum value, then $R$ and ALLD are bistable. This means that those choose between ALLD and $R$ will be the most beneficial.

### 2.4. Incentive mechanism analysis

A dynamic replicator describes a dynamic selection process. There are many different dynamic models in a developing process. A widely used dynamic replicator model was given by Taylor and Jonker in 1978. Just as the "survival of the fittest" [13] in biology, there are two stages existing in the evolutionary game: a selection mechanism and a mutation mechanism [14]. The selection mechanism denotes that a strategy which wins higher fitness in a game will be selected by more nodes in the next game. The mutation mechanism denotes that the nodes select the strategies randomly. If the mutation strategy wins the game, it will go on, otherwise it will be eliminated. The description of evolutionary game dynamics is the replicator equation which is a nonlinear differential equation that describes how the frequencies of a certain strategy changes over time. The differential equation is given as follows:

$$\frac{dx_i}{dt} = [u(s_i, x) - u(x, x)] \tag{3}$$

where $u(s_i, x)$ denotes the fitness which the populations gains from the pure strategy $s_i$ in random fitness among individuals of populations; $u(x, x)$ denotes the average fitness of populations; $u(x, x) = \sum_{i=1}^{k} x_i u(s_i, x)$, where $k$ is the number of pure strategies.

Eq. (3) denotes that if the fitness of a node with strategy $i$ gain is more than the average fitness, the growth rate of the populations who selected the $i$ strategy is positive. If the fitness of a node with strategy $i$ gain is less than the average fitness, the growth rate of the populations who selected the $i$ strategy is negative. If the fitness of a node with strategy $i$ gain equals the average fitness, the growth rate of the populations who selected strategy $i$ is 0.

We assume that the rate of the nodes in wireless sensor networks employing strategies ALLC, ALLD and $R$ are $x_{\mathrm{ALLC}}$, $x_{\mathrm{ALLD}}$ and $x_R$ respectively, where $x_{\mathrm{ALLC}}+x_{\mathrm{ALLD}}+x_R = 1$. For simplicity, we use strategies 1, 2, 3 instead of strategies ALLC, ALLD and R. Therefore, $x_{\mathrm{ALLC}}, x_{\mathrm{ALLD}}, x_R$ change into $x_1, x_2, x_3$. The strategy distribution of the population at a moment can be denoted as $X = (x_1, x_2, x_3)$. According to Eq. (3), we can induce the dynamic replicator equation of incentive mechanism:

(1) The expected fitness of node $i$:

$$U_i(x) = \sum_{j=1}^{3} x_j a_{ij} = (AX)_i. \tag{4}$$

(2) The average fitness of the population is :

$$\overline{U}_{(X)} = \sum_{i=1}^{3} x_i U_i(x) = X \cdot AX. \tag{5}$$

(3) Deterministic evolutionary game dynamics are given by the replicator equation:

$$\frac{dx_i}{dt} = (U_i(x) - \overline{U}_i(X))x_i = ((AX)_i - X \cdot AX)x_i. \tag{6}$$

The replicator equation denotes that the nodes with a specific strategy whose fitness is better than the average will shrink, conversely, they will grow.

According to the above analysis, we can calculate the dynamic replicator equation of every strategy:

(1) The expected fitness of strategy 1 (ALLC):

$$U_1(X) = (x_1 + x_3)R - (x_1 + x_2 + x_3)C. \tag{7}$$

(2) The expected fitness of strategy 2 (ALLD):

$$U_2(X) = x_1 R. \tag{8}$$

(3) The expected fitness of strategy 3 (R):

$$U_3(X) = (x_1 + x_3)R - (x_1 + x_3)C - (x_1 + x_2 + x_3)C_R. \tag{9}$$

(4) The average fitness of the population:

$$\overline{U}(X) = [(x_1 + x_2)^2 + x_1x_2]R - [(x_1 + x_3)^2 + x_1x_2]C - (x_1x_3 + x_2x_3 + x_3^2)C_R. \tag{10}$$

(5) The dynamic replicator equation of strategy 1 (ALLC):

$$\begin{aligned}\frac{dx_1}{dt} &= [x_1^2 + x_1x_3 - x_1(x_1 + x_3)^2 - x_1^2x_2]R + [x_1(x_1 + x_3)^2 + x_1^2x_2 - x_1^2 - x_1x_2 - x_1x_3]C \\ &\quad + (x_1^2x_3 + x_1x_2x_3 + x_1x_3^2)C_R.\end{aligned} \tag{11}$$

(6) The dynamic replicator equation of strategy 2 (ALLD):

$$\frac{dx_2}{dt} = [x_1x_2 - x_2(x_1 + x_3)^2 - x_1x_2^2]R + [x_2(x_1 + x_3)^2 + x_1x_2^2]C + (x_2^2x_3 + x_1x_2x_3 + x_2x_3^2)C_R. \tag{12}$$

(7) The dynamic replicator equation of strategy 3 (R):

$$\begin{aligned}\frac{dx_3}{dt} &= [x_1x_3 + x_3^2 - x_3(x_1 + x_3)^2 - x_1x_2x_3]R + [x_3(x_1 + x_3)^2 + x_1x_2x_3 - x_1x_3 - x_3^2]C \\ &\quad + (x_1x_3^2 + x_2x_3^2 + x_3^2 - x_1x_3 - x_2x_3 - x_3^2)C_R.\end{aligned} \tag{13}$$

Now let us consider the traditional evolutionary game dynamics of three strategies given by the replicator equation. This approach describes the deterministic selection in an infinitely large population. The frequency of a strategy increases at a rate given by the difference between its fitness and the average fitness of the population. The fitness of a strategy is the expected payoff of the game which assumes many random encounters with other individuals. In this framework, any mixed population of ALLC, $R$, and ALLD will converge to a pure ALLD population. The state where everybody plays ALLD is the only stable equilibrium.

## 3. Stability analysis

### 3.1. Stochastic dynamics in finite populations

Assume that the size of the population is N and let $u \geq 0$ denote the mutation probability. Let $A = (a_{ij})_{i,j=1}^{3}$ be a positive payoff matrix. We define a frequency dependent Moran [15–17] process $X(t) = X(t; u, N, A)$, $t = 0, 1, 2, \dots,$ on the state space

$$S_N = \{(x_1, x_2, x_3) \in {N_0}^3 : x_1 + x_2 + x_3 = N\}.$$

Here $x_i$ denotes the number of nodes using strategy $i$. If the strategy of nodes is in state $(x_1, x_2, x_3)$, then the payoff of the nodes using strategy $i$ is

$$F_i = \sum_{j=1}^{3} a_{ij}x_j - a_{ii}.$$

The reason why we subtract $a_{ii}$ is that the node cannot interact with itself. The probability that the offspring of this individual will use strategy $i$ is $1 - 2u$. Here, $u$ denotes the probability that the nodes who adopt strategy $i$ will change its strategy into $j$. With probability $u$, the offspring will use one of the two other strategies. If $u > 0$, then the matrix will be irreducible. Thus, we can define a stationary distribution which is determined by the left-hand eigenvector $\pi = \pi(s; u, N, A)$, $s \in S_N$.

### 3.2. Evolutionary stable strategy

Evolutionary stable strategy (ESS) is a strategy, if adopted by a population of players which cannot be invaded by any alternative strategy that is initially rare. An ESS is an equilibrium refinement of the Nash equilibrium. A Nash equilibrium which is evolutionary stable means that once it is fixed in a population, natural selection alone is sufficient to prevent mutant strategies from successfully invading.

The ESS was developed in order to define a class of solutions to game theoretic problems, which equivalent to the Nash equilibrium, and could be applied to the evolution of social behavior in animals. Nash equilibria may sometimes exist due to the application of rational foresight, which would be inappropriate in an evolutionary context. Teleological forces such as rational foresight cannot explain the outcomes of trial-and-error processes, such as evolution, and thus have no place in biological applications. The definition of an ESS excludes such Nash equilibria.

Strategy $k$ is ESS if either (I) $a_{kk} > a_{ik}$ or (II) $a_{kk} = a_{ik}$ and $a_{ki} > a_{ii}$ holds for all $i \neq k$.

**Lemma 1** (*[18]*)**.** *When the reciprocation cost limits to zero, for a fixed number of peers in network system and a very small mutation probability, the time average of these oscillations can be mostly dominated by ALLC and R.*

Hence, we have the corollary by Lemma 1.

**Corollary 1.** *When the reciprocation cost limits to zero, for a fixed number of nodes in the wireless sensor network and a very small mutation probability, for most of the time, the population is in a state that consists of ALLC and R nodes.*

**Theorem 1.** *With small reciprocation cost and the mutation probability limited to zero, ALLD is the only Nash equilibrium in the incentive mechanism. That is, any mixed population of ALLC, ALLD and R will finally converge to a full ALLC population.*

**Proof.** We assume that a game has $n$ strategies. The payoff values are given by an $n \times n$ payoff matrix $A = [a_{ij}]$. Strategy $k$ is a strict Nash equilibrium if $a_{kk} > a_{ik}$ for all $i \neq k$. Strategy $k$ is a Nash equilibrium if $a_{kk} \geq a_{ik}$, for all $i$.

The above definition implies that, if strategy $k$ is a strict Nash equilibrium, then it is an ESS; if $k$ is an ESS, then it is a Nash equilibrium. Both Nash equilibrium and ESS give conditions on whether a strategy which is played by the majority of players outperforms all the other strategies. □

Through the theorem, we can obtain several inferences as follows:

(1) When only ALLC and ALLD users exist in a population, after the evolutionary dynamics for a period of time, all the nodes will lead to ALLD fully in a wireless sensor network. Finally, full ALLD will exist and the population will keep this stable state for a period of time.
(2) When only ALLC and $R$ users exist in a population, after the evolutionary dynamics for a period of time, all the nodes will lead to ALLC fully in wireless sensor network. Because of the existence of mutations, all ALLD populations will take over the ALLC population. Finally, full ALLD will exist and keep this stable state for a period of time.

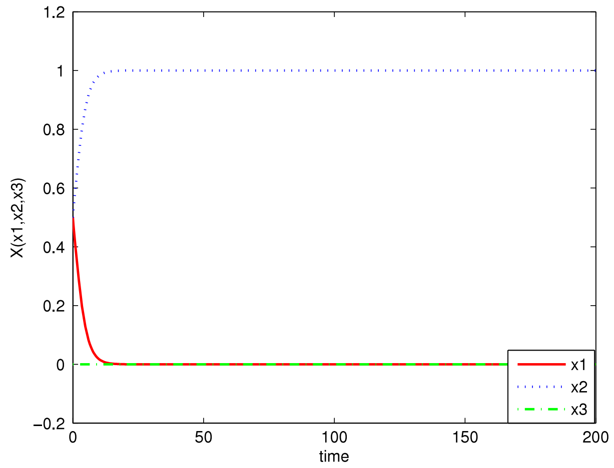


**Fig. 1.** The state of population when $C_R = 0.1, X = [\frac{1}{2}, \frac{1}{2}, 0]$.

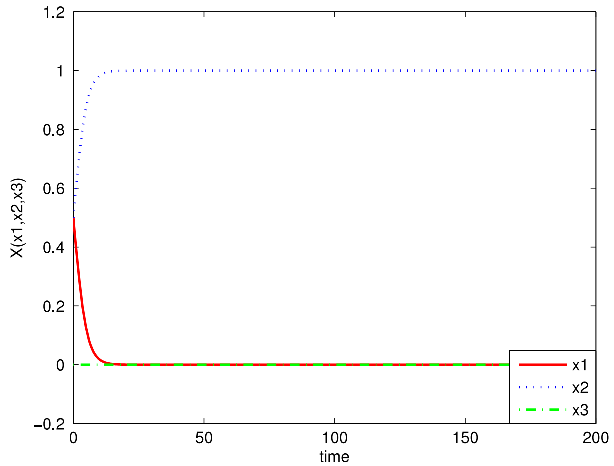


**Fig. 2.** The state of population when $C_R = 0.1, X = [\frac{1}{2}, \frac{1}{2} - \frac{1}{10000}, \frac{1}{10000}]$.

(3) When only ALLD and $R$ users exist in a population, and if the frequency of $R$ satisfies the inequality $x_3 > \frac{C_R}{R-C}$, after the evolutionary dynamics for a period of time, the evolutionary dynamics will lead to $R$ fully. Because of the existence of mutations, a single mutant ALLC user will take over the $R$ population. Finally, full ALLD will quickly replace the ALLC population and the population will last in this stable state for a period of time. If the frequency of $R$ satisfies the equality $x_3 = \frac{C_R}{R-C}$, after the evolutionary dynamics for a period of time then the evolutionary dynamics will lead to $R$ fully. Because of the existence of mutations, full ALLD will quickly replace the $R$ population and the population will keep this stable state for a period of time. If the frequency of $R$ satisfies the equality $x_3 = \frac{C_R}{R-C}$, after the evolutionary dynamics for a period of time, the whole population will be finally stuck in full ALLD and the population can resist small mutations and keep this stable state for a period of time.

## 4. Performance evaluation

The mutation probability and reciprocation cost both make a big difference between the population strategies. Some previous works have been given in [18]. In this paper, we turn to simulations for estimating the state of the population. Since the different measurement criteria of parameters such as the fitness and the cost of the nodes, we will normalize all the parameters, the values distributed between 0 and 1, fixed parameters for simulation are listed as $R = 1.0$, $C = 0.4$. According to the replicator equations of ALLC, ALLD and $R$, we can build a simulation model.

(1) When $C_R = 0.1$, initial $X = [\frac{1}{2}, \frac{1}{2}, 0]$ and $[\frac{1}{2}, \frac{1}{2} - \frac{1}{10000}, \frac{1}{10000}]$ respectively, the state of the population is shown in Figs. 1 and 2.

Fig. 1 shows that after a period of the evolutionary game, the nodes in the wireless sensor network will choose the ALLD strategy. Fig. 2 shows that the population can resist a small mutation and the population will remain this stable state.

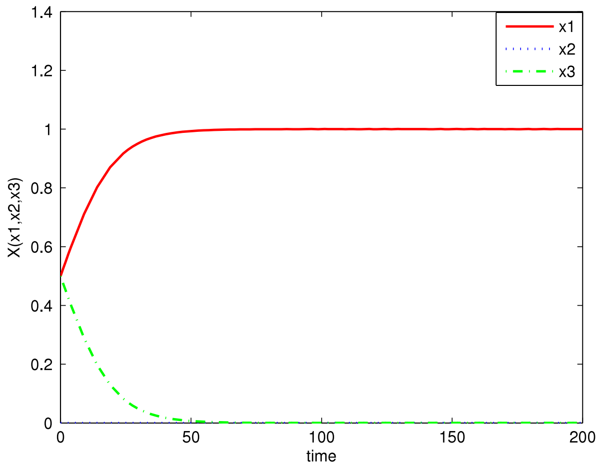


**Fig. 3.** The state of population when $C_R = 0.1, X = [\frac{1}{2}, 0, \frac{1}{2}]$.

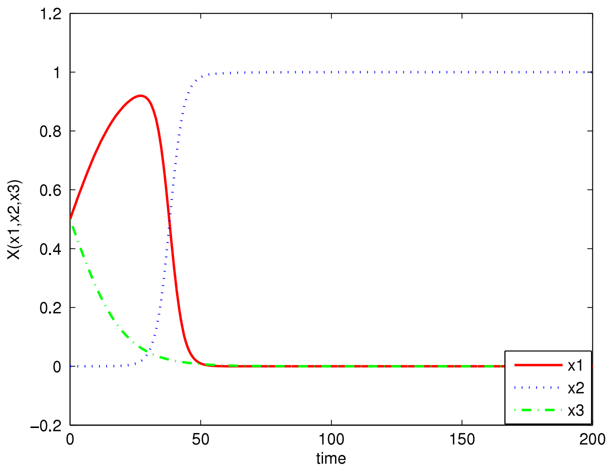


**Fig. 4.** The state of population when $C_R = 0.1, X = [\frac{1}{2} - \frac{1}{10000}, \frac{1}{10000}, \frac{1}{2}]$.

(2) When $C_R = 0.1$, initial $X = [\frac{1}{2}, 0, \frac{1}{2}]$ and $[\frac{1}{2} - \frac{1}{10000}, \frac{1}{10000}, \frac{1}{2}]$ respectively, the state of the population is shown in Figs. 3 and 4.

Fig. 3 shows that after a period, the nodes in the wireless sensor network will choose the ALLC strategy. Fig. 4 shows that if there is only a small mutation of the nodes, the nodes who choose strategy ALLC will choose strategy ALLD at last. In the above figures, we can get that the dynamics forms endless oscillation from ALLD to $R$ to ALLC and back to ALLD in the limiting case of a very small mutation rate and under specific conditions. That is, the population is almost homogeneous for one strategy for a long time, but then a mutant generates a lineage which takes over the population. However, this does not mean that all of these states are equally plausible, as they may differ in their robustness to a small probability of mutation or experimentation. There is one way to formalize the idea that some of the homogeneous states are "more" persistent than others is to assume that a small mutation term makes the system stable, and then analyze the limitation of the invariant distribution, as the mutation probability goes to zero.

(3) When $C_R = 0.1$, initial $X = [0, \frac{1}{2}, \frac{1}{2}]$ and $[\frac{1}{10000}, \frac{1}{2}, \frac{1}{2} - \frac{1}{10000}]$ respectively, the state of the population is shown in Figs. 5 and 6.

Fig. 5 shows that the nodes in the wireless sensor network will choose the $R$ strategy after a period of the evolutionary game. Fig. 6 shows that if there is only a small mutation of nodes, the nodes who choose strategy $R$ will choose strategy ALLC, and then they all choose ALLD, making all the nodes of the population choose the strategy ALLD at last.

(4) When $C_R = 0.1$, initial $X = [0, \frac{5}{6}, \frac{1}{6}]$ and $[\frac{1}{10000}, \frac{5}{6}, \frac{1}{6} - \frac{1}{10000}]$ respectively, the state of the population is shown in Figs. 7 and 8.

Fig. 7 shows that the nodes who choose strategy ALLD and $R$ exist at the same time. Fig. 8 shows that if there is only a small mutation of the nodes, the nodes who choose strategy $R$ will choose strategy ALLD at last.

(5) When $C_R = 0.1$, initial $X = [0, \frac{6}{7}, \frac{1}{7}]$ and $[\frac{1}{10000}, \frac{6}{7} - \frac{1}{10000}, \frac{1}{7}]$ respectively, the state of the population is shown in Figs. 9 and 10.

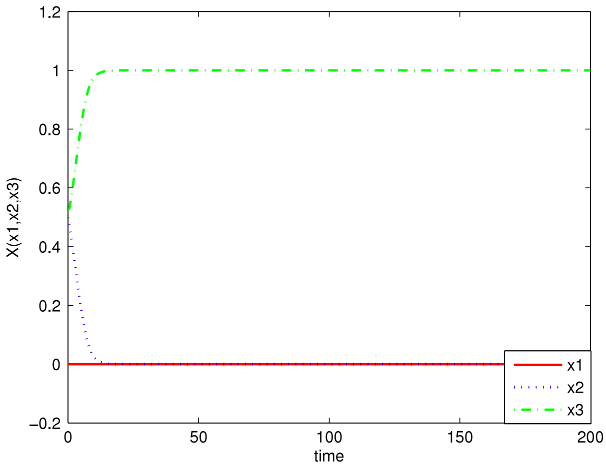


**Fig. 5.** The state of population when $C_R = 0.1, X = [0, \frac{1}{2}, \frac{1}{2}]$.

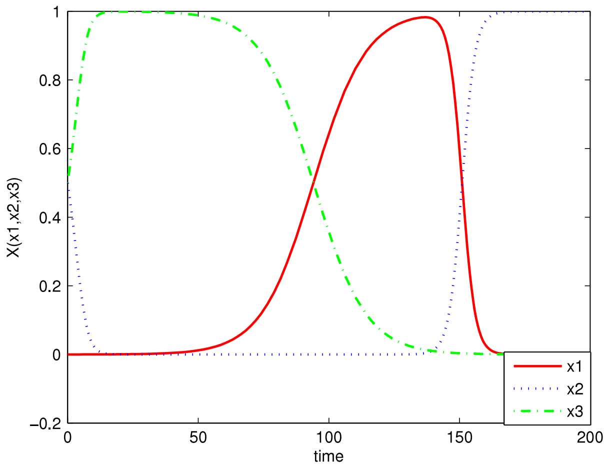


**Fig. 6.** The state of population when $C_R = 0.1, X = [\frac{1}{10000}, \frac{1}{2}, \frac{1}{2} - \frac{1}{10000}]$.

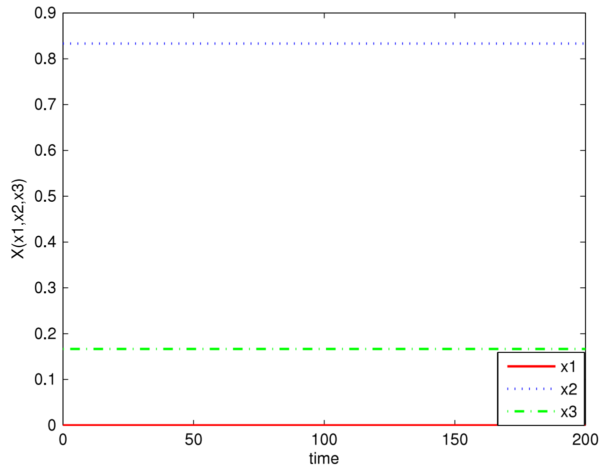


**Fig. 7.** The state of population when $C_R = 0.1, X = [0, \frac{5}{6}, \frac{1}{6}]$.

Fig. 9 shows that after a period of the evolutionary game, the nodes in the wireless sensor network will choose the ALLD strategy. Fig. 10 shows that the population can resist a small mutation and the population will remain this stable

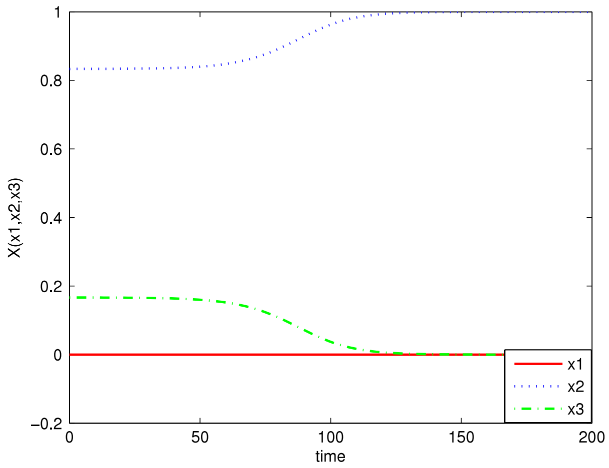


**Fig. 8.** The state of population when $C_R = 0.1, X = [\frac{1}{10000}, \frac{5}{6}, \frac{1}{6} - \frac{1}{10000}]$.

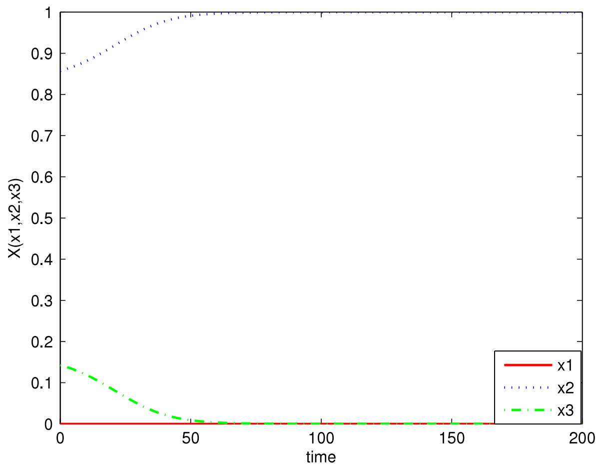


**Fig. 9.** The state of population when $C_R = 0.1, X = [0, \frac{6}{7}, \frac{1}{7}]$.

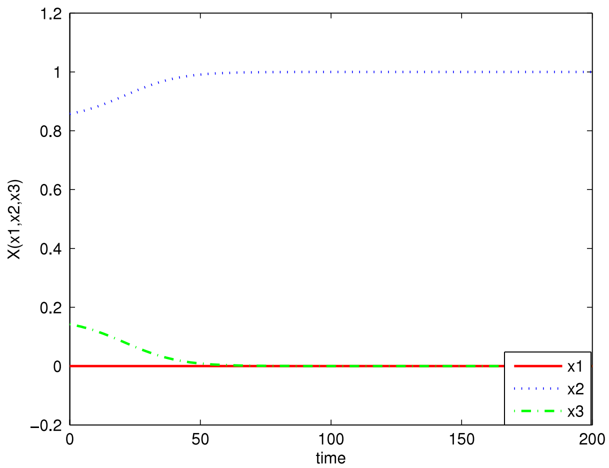


**Fig. 10.** The state of population when $C_R = 0.1, X = [\frac{1}{10000}, \frac{6}{7} - \frac{1}{10000}, \frac{1}{7}]$.

state. Note that when there exist only ALLD and $R$ in population, and $x_R > \frac{c_R}{\alpha - 1}$ then the natural selection would favor R. Otherwise, all favor ALLD. According to the experimental parameters, it is required $x_R > \frac{0.1}{5-1} = 0.025$ for $R$ to probably

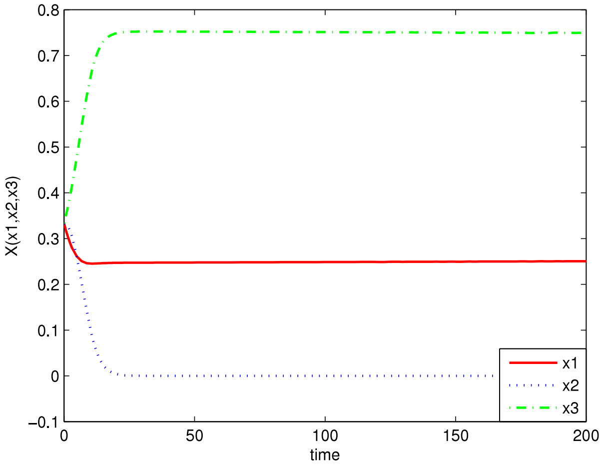


**Fig. 11.** The state of population when $C_R = \frac{1}{10000}$, $X = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$.

take over ALLD. Thus, when the mutation probability is limited to zero, the whole population will finally converge to full ALLD. The difference between strong selection and weak selection is that, the latter will take more time to get to the full ALLD state.

(6) When $C_R = \frac{1}{10000}$, initial $X = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$, the state of the population is shown in Fig. 11.
Fig. 11 shows that when the cost of strategy $R$ limited to 0 nor is the probability of mutation very small, then only the nodes who choose strategy ALLD or R exist. However, the number of nodes who choose strategy $R$ is more than the number of those who choose ALLC. Note that the results in Fig. 11 are quite different from the results obtained in [19,20], in which the whole population will converge to full $R$ or be located in the edge between $R$ and ALLC when reciprocation cost is zero and there no mutations exist. When considering a little more realistic networked environment (that is, $R$ adopter faces extra mutation probability to mimic the error in nodes learning process), ALLD is the only equilibrium.

## 5. Conclusion

In this paper, we adopt the evolutionary game to model the dynamics of a reciprocity-based incentive mechanism in which there are three information seeking alternative strategies: ALLC, ALLD and $R$. Firstly, according to the character, cost, reciprocation cost and small mutation in the wireless sensor network, we establish an incentive game model of the nodes for forwarding packages. Secondly, we analyze the dynamics and stability of the incentive game model, emphasizing that the nodes adjust themselves to find the most suitable strategies through studying, imitating and "trial and error". By doing these to make the network reach a cooperative state. Finally, we perform simulation experiments to validate the correctness of our theoretical analyses. In this model, there are two special points which should be taken into consideration. One is the small reciprocative cost which is used for seeking information for $R$ users, the other is the mutation-selection. Mutation means that the offspring strategy does not adopt the same strategy as their parent, but one of the two other strategies even though the mutation probability is limited to zero. Thinking another way, the mutation may be defined as mistakes or innovation when nodes imitate others' strategies.

Through Nash equilibrium analysis, we infer that only the ALLD is a strict Nash equilibrium. We can gain the state of population obviously with different reciprocative cost and different ratios of each strategy nodes by the simulation tool. At last, we theoretically analyze that the ratio of each strategy and the reciprocative cost $C_R$ plays an important role in the evolutionary dynamics of incentive mechanism. According to the analyses, we should take relative, active and defective measures in wireless sensor networks.

As the communications systems change into highly decentralized and self-organizing networks, we can find that there are great applications in the field based on the evolutionary game theory analysis. Especially, they are widely used in wireless networks. Considering the selfish nodes in the wireless sensor networks, we should analyze the situation and take actions to make each selfish node cooperate with others and then provide the normal service for the whole network. One way to combat rational behaviors in such networks is to model mechanisms (just like the incentive mechanism) to stimulate cooperation among nodes. Our research is the first attempt to analyze the evolutionary dynamics of soft security mechanisms based on the evolutionary game theory.

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